Lec 10 digital Control

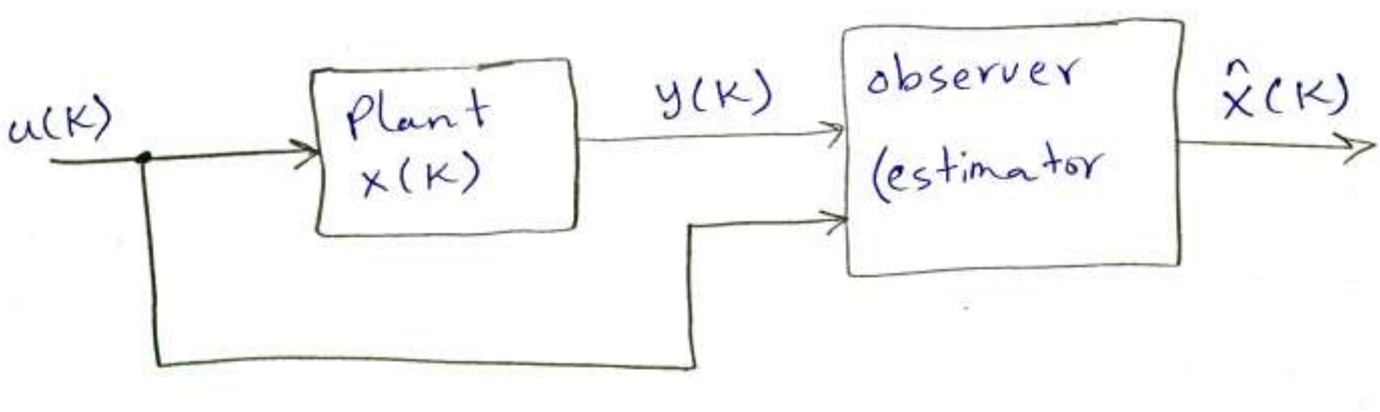
⇒ observer Design

→ for state feedback Control (Pole Placement design)

$$u(K) = -K\ x(K)$$

→ It is required for all states to be accessible for measuring:

⇒ In this Case of some or all of the states are not accessible for measuring, an observer should be designed to estimate these states values. The estimation is done by the history of data for i/P and o/P.



⇝ The observer equation:-

$$\hat{x}(K+1) = (A - GC)\,\hat{x}(K) + Bu(K) + G\,y(K)$$

Where:

$G \longrightarrow$ gain matrix

$$\text{gain matrix } G = \begin{pmatrix} g_1 \\ g_2 \\ \vdots \\ g_n \end{pmatrix}$$

Where $n \longrightarrow$ order of the system

⇒ The gain matrix is determined through the specs of the observer as:

* observer speed response
* transient
* settling time

$\left.\right\}$ $\zeta, \omega_n \rightarrow$ desired poles of observer

⇝ The gain matrix G is determined by:-

① desired ch. equation for the observer "$\alpha_o(z)$"

$$\alpha_o(z) = (z-P_1)(z-P_2)\dots(z-P_n) \quad \text{so} \ ①$$

where $P_1, P_2, \dots P_n$ are desired Poles

$$\hat{x}(k+1) = (A-Gc)\hat{x}(k) + Bu(k) + G\,y(k)$$

The observer ch-equation:

$$|zI - A + Gc| = 0 \longrightarrow ②$$

Compare ① & ② to get G

$\boxed{2}$ using Ackermann's method

$$G = \begin{bmatrix} g_1 \\ g_2 \\ \vdots \\ g_n \end{bmatrix} = \alpha_o(A)\, M_o^{-1} \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{pmatrix}$$

For $n=2$

$$G = \begin{bmatrix} g_1 \\ g_2 \end{bmatrix} = \alpha_o(A)\, M_o^{-1} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

where $\mathcal{M}_o \equiv$ observability matrix

$$\alpha_o(A) = \alpha_o(z)\Big|_{z=A}$$

$\alpha_o(z) \rightsquigarrow$ desired ch.-equation for observer.

Ex $x(K+1) = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} x(K) + \begin{pmatrix} 2 \\ 2 \end{pmatrix} u(K)$

$y(K) = \begin{pmatrix} 1 & 0 \end{pmatrix} x(K)$

(1) Determine the gain matrix K such that desired closed loop poles located at:

$$Z_{1,2} = 0.528 \pm J\,0.295$$

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$$K = \begin{pmatrix} 0.0775 & 0.3945 \end{pmatrix}$$

$$K = \begin{pmatrix} 0 & 1 \end{pmatrix} \mathcal{M}_c^{-1} \alpha_c(A)$$

$$\alpha_c(z) = z^2 - 1.056z + 0.366$$

$$\alpha_c(A) = \begin{pmatrix} 0.31 & 1.888 \\ 0 & 0.31 \end{pmatrix}$$

$$\mathcal{M}_c^{-1} = \begin{pmatrix} -0.25 & 0.75 \\ 0.25 & -0.25 \end{pmatrix}$$

(2) Design a full order observer such that $\zeta = 1$ & the time constant of the observer is one half the time constant of the desired poles in (1), $T = 1\,sec$

$$\tau_o = \frac{1}{2}\tau_c \rightarrow (1)\ ?$$

للتوضيح

$$S_{1,2} = -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2}$$

$$\hookrightarrow Z_{1,2} = r\angle \pm\theta = r\cos\theta \pm j\sin\theta$$

$$r = e^{-\zeta\omega_n T} = e^{-T/\tau}$$

$$\theta = \pm\omega_n T\sqrt{1-\zeta^2} \quad ; \quad \tau = \frac{1}{\zeta\omega_n}$$

$$r = \sqrt{(0.528)^2 + (0.295)^2} = 0.605$$

$$r = e^{\frac{-T}{\tau_c}} \Rightarrow \ln r = -\frac{T}{\tau}$$

$$\ln(0.605) = -\frac{1}{\tau_c} \Rightarrow \boxed{\tau_c \approx 2\ sec}$$

$$\tau_o = \frac{1}{2}\tau_c$$

$$\tau_o = \frac{1}{2} * 2 = 1 \text{ sec}$$

→ The desired Poles of the observer:

$\tau_o = 1\,\text{sec}$ & $\zeta = 1$

$$\tau_o = \frac{1}{\omega_n * \zeta} \Rightarrow \omega_n = 1$$

$$r = e^{-\zeta \omega_n T} = e^{-\frac{T}{\tau_o}} = e^{-1} = 0.3678$$

$$\theta = \omega_n T \sqrt{1-\zeta^2} = 0$$

$$Z_{1,2} = r\cos\theta \pm J\,\underbrace{r\sin\theta}_{\to 0} = r = 0.3678$$

→ The desired ch. equation for the observer $\alpha_o(z)$:

$$\alpha_o(z) = (Z - 0.3678)^2$$

$$\alpha_o(z) = Z^2 - 0.7356\,Z + 0.1353$$

$$\alpha_0(A) = A^2 - 0.7356\,A + 0.1353\,I$$

$$= \begin{pmatrix} 0.3997 & 2.5288 \\ 0 & 0.3997 \end{pmatrix}$$

$$G = \alpha_0(A)\, M_0^{-1} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} C \\ CA \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 1 & 2 \end{pmatrix}$$

$$M_0^{-1} = \begin{pmatrix} 1 & 0 \\ -0.5 & 0.5 \end{pmatrix} \Rightarrow G = \begin{pmatrix} 1.2644 \\ 0.1998 \end{pmatrix}$$

Ex2: $$x(K+1) = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} x(K) + \begin{pmatrix} 1 \\ 1 \end{pmatrix} u(K)$$

$$y(K) = \begin{pmatrix} 1 & 0 \end{pmatrix} x(K)$$

→ Design a full order observer such that:

a) time constant is 10 sec

b) desired observer poles must be real and equal $\Rightarrow$ $\boxed{\zeta = 1}$

$T = 1\ sec$; $s_{1,2} = -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2}$

* Desired poles of observer

$$r = e^{-\frac{T}{\tau}} = e^{-\frac{1}{10}} = 0.905$$

$$\Theta = \omega_n T \sqrt{1-\underbrace{\zeta^2}_{1}} = 0 \quad \text{(Real and equal)}$$

$$Z_{1,2} = 0.905$$

→ The desired ch. eq. for the observer $\alpha_o(z)$:

$$\alpha_o(z) = (z - 0.905)^2 = z^2 - 1.8z + 0.819$$

$$\alpha_o(A) = \begin{pmatrix} 0.009 & 0.19 \\ 0 & 0.009 \end{pmatrix}$$